

Topic: Horizontal and vertical shifts

Question: Which of the following tables shows the correct values of the function?

$$-\sin\left(\theta - \frac{\pi}{6}\right)$$

Answer choices:

A

θ	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$-\sin\left(\theta - \frac{\pi}{6}\right)$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$

B

θ	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$-\sin\left(\theta - \frac{\pi}{6}\right)$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$

C

θ	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$-\sin\left(\theta - \frac{\pi}{6}\right)$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$

D

θ	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$-\sin\left(\theta - \frac{\pi}{6}\right)$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1



Solution: C

For the time being, let's ignore the overall minus sign in the function $-\sin(\theta - (\pi/6))$, and tabulate the values of $\theta - (\pi/6)$ and $\sin(\theta - (\pi/6))$ at the indicated angles θ :

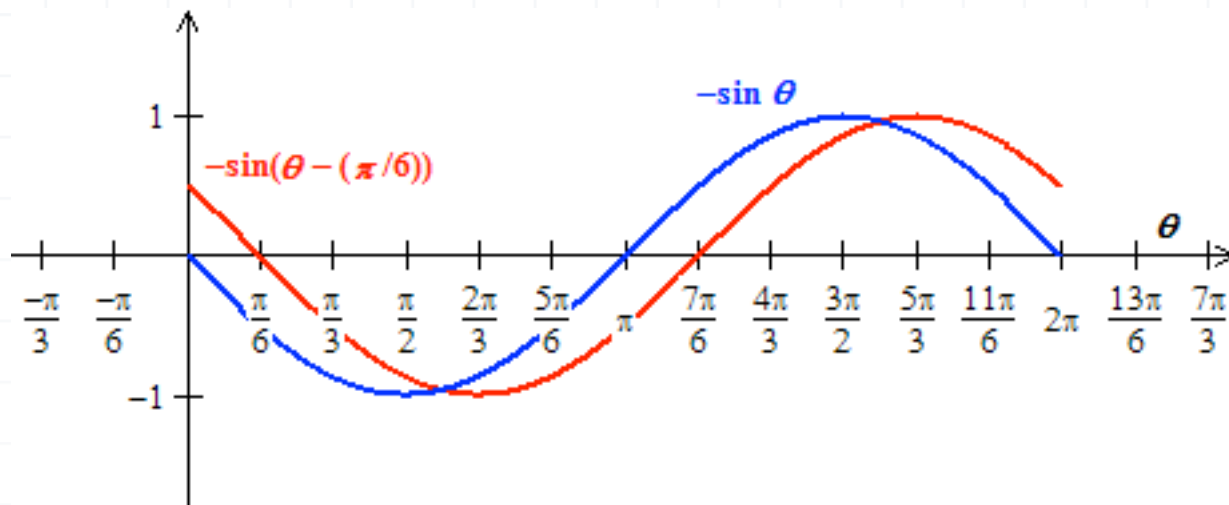
θ	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$\theta - \frac{\pi}{6}$	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$
$\sin \left(\theta - \frac{\pi}{6} \right)$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$

Now all that remains is to multiply each of those values by -1 , to take care of the overall minus sign.

θ	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$-\sin \left(\theta - \frac{\pi}{6} \right)$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$

To visualize the shift, we'll graph the functions $-\sin \theta$ and $-\sin(\theta - (\pi/6))$ over the interval $[0,2\pi)$.





If we express $-\sin(\theta - (\pi/6))$ as $-\sin(\theta + c)$, then $c = -\pi/6$ (hence negative). Thus we expect the graph of $-\sin(\theta - (\pi/6))$ to be shifted $\pi/6$ units to the right (relative to the graph of $-\sin \theta$), and that is precisely what we see.



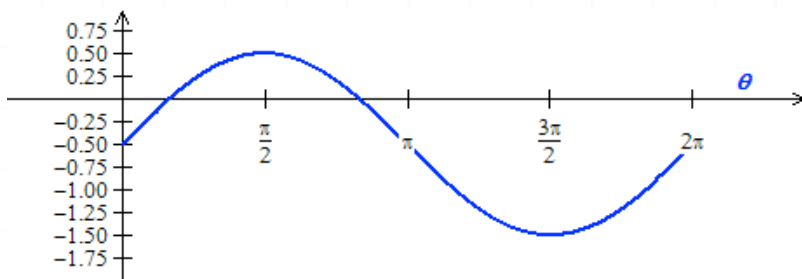
Topic: Horizontal and vertical shifts

Question: Which of the following curves is the graph of the function?

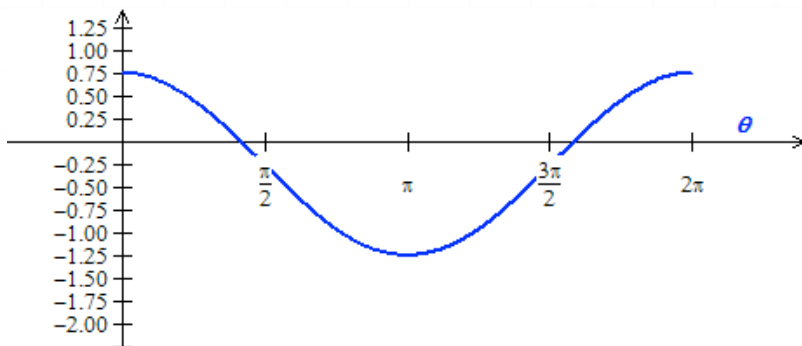
$$\cos \theta - 0.25 \text{ over the interval } [0, 2\pi)$$

Answer choices:

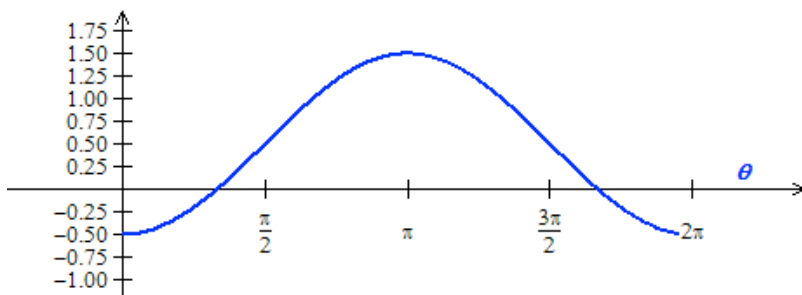
A



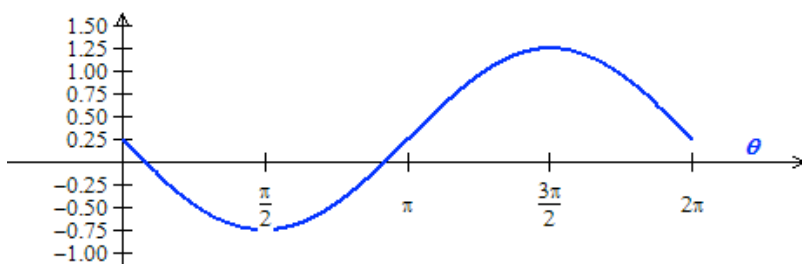
B



C



D



Solution: B

The function $\cos \theta - 0.25$ is in the form $\cos \theta + d$, with $d = -0.25$. Therefore, the values of this function range from $-1 + (-0.25) (= -1.25)$ to $1 + (-0.25) (= 0.75)$. This rules out all the curves other than the one shown in answer choice B.



Topic: Horizontal and vertical shifts**Question:** Which of the following is accurate?

Consider the following statements:

I The graph of the function $\sin(\theta - (\pi/2))$ over the interval $[0, 2\pi)$ is identical to the graph of the function $\cos \theta$ over that interval.

II The graph of the function $\sin(\theta - (\pi/2))$ over the interval $[0, 2\pi)$ is identical to the graph of the function $-\cos \theta$ over that interval.

Answer choices:

- A Statement I is true, and statement II is false.
- B Statement I is false, and statement II is true.
- C Statements I and II are both true.
- D Statements I and II are both false.



Solution: B

Answer choice C can be eliminated, because if statements I and II were both true, then the graph of the function $\cos \theta$ over the interval $[0, 2\pi)$ would be identical to the graph of the function $-\cos \theta$ over that interval, which is clearly not the case. We will show that statement II is true (hence that statement I is false).

First, we'll compare the values of $\sin(\theta - (\pi/2))$ and $-\cos \theta$ at “key angles” θ in the interval $[0, 2\pi)$:

θ	$\theta - \frac{\pi}{2}$	$\sin(\theta - \frac{\pi}{2})$	$\cos \theta$	$-\cos \theta$
0	$-\frac{\pi}{2}$	-1	1	-1
$\frac{\pi}{2}$	0	0	0	0
π	$\frac{\pi}{2}$	1	-1	1
$\frac{3\pi}{2}$	π	0	0	0

To verify that $\sin(\theta - (\pi/2)) = -\cos \theta$ at every angle θ in the interval $[0, 2\pi)$, we can apply the difference identity for sine to the angles θ and $\pi/2$:

$$\sin\left(\theta - \frac{\pi}{2}\right) = \sin \theta \left(\cos \frac{\pi}{2}\right) - \cos \theta \left(\sin \frac{\pi}{2}\right)$$

$$\sin\left(\theta - \frac{\pi}{2}\right) = \sin \theta(0) - \cos \theta(1)$$



$$\sin \left(\theta - \frac{\pi}{2} \right) = -\cos \theta$$

